Aggregation and Design of Information in Asset Markets with Adverse Selection

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Since Hayek (1945), information aggregation has been an important topic in economics and finance.

- Typically, analyzed in the literature in the context of centralized markets for homogenous assets.

This paper: study information aggregation in the context of decentralized markets for heterogeneous but correlated assets.

1. Does the aggregate trading behavior in the market reveal the underlying fundamentals?
2. Is laissez-faire equilibrium efficient, i.e., is there scope for an optimal design of disclosure policies?
Introduction

Our framework:

- Many sellers ($N$), each privately informed about quality of her asset.
- Buyers compete, are uninformed, and face a lemons problem à la Akerlof.
- Asset qualities are correlated with an unobservable aggregate state of nature, i.e., there are more bad assets in the low state.
- Trade occurs over time. Key feedback:

  (Expected) info arrival $\implies$ Trading behavior $\implies$ (Actual) info arrival

Our objective: study a large market ($N \uparrow \infty$) and its information properties from both positive and normative perspectives.
Sellers

- There are two trading dates, $t = 1, 2$.
- There are $N$ sellers, $i = \{1, ..., N\}$. Each seller owns an indivisible asset.
- Each seller is privately informed about the type of her asset, denoted by $\theta_i \in \{L, H\}$. Seller values asset of type $\theta$ at $c_\theta$, with $c_L < c_H$.
- The payoff to a seller with asset of type $\theta$ who agrees to trade at price $p$ at time $t$ is:
  \[(1 - \delta^{t-1})c_\theta + \delta^{t-1}p\]
  where $\delta$ is the discount factor. If the seller never trades, her payoff is $c_\theta$. 
Each seller has multiple potential trading partners or “buyers.” Buyers value asset of type $\theta$ at $v_\theta$, with $v_L < v_H$.

There is common knowledge of gains from trade: $v_\theta > c_\theta$.

Each period, given their information, buyers bid for the assets.

The payoff to a buyer who purchases an asset of type $\theta$ at price $p$ is:

$$v_\theta - p$$

If the buyer does not trade, his payoff is 0.
Uncertainty and correlation

There is an unobserved state of nature $S \in \{l, h\}$:

- Unconditional distribution is:
  \[
  \mathbb{P}(\theta_i = L) = \mathbb{P}(S = l) = 1 - \pi_0.
  \]

- Conditional distribution of asset types is:
  \[
  \mathbb{P}(\theta_i = L | S = l) = \lambda > 1 - \pi_0.
  \]
Two assumptions

Assumption 1 (Lemons Condition) \( \pi_0 v_H + (1 - \pi_0) v_L < c_H \).

Assumption 2 (No Separation) \( v_L < (1 - \delta) c_L + \delta v_H \).

These assumptions rule out the first-best efficient outcome and ensure that dynamic considerations are relevant.
Equilibrium
Equilibrium structure

We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept.

1. Buyers’ offers are optimal given the seller’s strategy and other buyers’ strategy.
2. Seller’s optimize given the other seller’s strategy and expected offers from buyers.
3. Beliefs are updated by Bayes Rule.
Equilibrium structure

Second (last) period is a static Akerlof:

1. Buyers’ update their beliefs, based on past trading information.
2. Buyers bid the expected value, conditional on seller acceptance.
3. Sellers accept/reject and game ends.

First period:

1. Buyers make low offer.
   - Due to Skimming Property + Lemons Condition.
2. High types do not trade; low types trade with probability \( \sigma \in (0, 1) \).
   - All equilibria turn out to be symmetric, i.e., \( \sigma_i = \sigma \ \forall i \).
   - No Separation Condition \( \implies \sigma < 1 \). Endogenous information \( \implies \sigma > 0 \).
Price function in the second period

Note: Price in the second period is non-linear in $\pi$ due to adverse selection.
Price function in the second period

\( \pi \) depends on own trading prob. \( \sigma_i \) which moves beliefs from \( \pi_0 \) to \( \pi^{INT} \) and then information arrival generates a distribution of posteriors.
Option value effect: information arrival $\Rightarrow$ high prices conditional on good news. Since $\pi_0 < \bar{\pi}$, this effect is strong (weak) when $\sigma_i$ is low (high).
Option value effect: information arrival $\implies$ high prices conditional on good news. Of course, both $\sigma_i$ and the distribution of news (through $\sigma_{-i}$) are endogenous.
Trading incentives in the first period

- Given the Lemons Condition only low types trade in the first period. Prior is too low to have the pooling offer attract the high type to sell.
  - No high types trades.
  - Low types must trade with positive probability.

- Given the No Separation Condition low types cannot trade with probability 1 otherwise the price would be too high the second period and they would regret having traded.

- In equilibrium, low type must be indifferent between accepting $v_L$ the first period or waiting to trade in the second period.
Trading incentives in the first period

In equilibrium, low type must be indifferent to trade in the first period:

\[ \nu_L = Q_L(\sigma) \equiv (1 - \delta)c_L + \delta \mathbb{E}_L \{ F_L(\pi) \} \]

where:

- \( Q_L \) is the low type’s continuation value,
- \( \pi \) is the buyers’ (random) posterior belief that the seller is a high type,
- \( F_L \) is the low type’s expected payoff as a function of belief \( \pi \).
  - Essentially, equals the second period asset price.
Equilibrium summary

Finding and equilibrium boils down to finding $\sigma$ such that:

$$v_L = Q_L(\sigma)$$

- Equilibrium exists.
- There might be multiple equilibria.

We want to study the information properties of the equilibria as the market size $N$ becomes large.
Information aggregation
Information aggregation

For a given $N$, let $p_N$ denote the buyers’ posterior belief that the state is $h$, upon observing trading behavior in the first period.

- **Note:** information revealed by second period trades is payoff irrelevant.

**Definition**

There is information aggregation along a given sequence of equilibria if

$$ p_N \to 1 \{ S = h \} \text{ as } N \to \infty \text{ in probability.} $$

Let $\sigma_N$ denote the equilibrium trading probability when market size is $N$.

- If $\sigma_N$ were uniformly bounded away from 0, information would aggregate:
  - In state $s$, the fraction of trades would converge to population mean $\sigma_N \cdot \mathbb{P}(\theta_i = L | S = s)$.
  - But what if $\sigma_N \to 0$?

As it turns out, neither of these two cases is pathological!
It is useful to consider a ‘fictitious’ economy where:

- Aggregate state $S$ is revealed before trade at $t = 2$.
  $\Rightarrow$ Seller $i$ does not care about other sellers’ trading behavior.
- Equilibrium same as with only one seller and exogenous information.

At $t = 2$, buyers update their beliefs about seller $i$ based on two pieces of info:

- Seller $i$ rejected trade at $t = 1$.
- Aggregate state is $S$. 
Fictitious economy

Lemma

The unique equilibrium of the fictitious economy involves zero probability of trade in the first period (i.e., $\sigma_i = 0$) if and only if

$$Q^{i,fict}_L |_{\sigma_i=0} \geq v_L, \quad (\ast)$$

which holds if and only if $\lambda$ and $\delta$ satisfy the following:

$$\lambda \geq \bar{\lambda} \equiv 1 - \frac{\pi(1 - \bar{\pi})}{1 - \pi}$$

and

$$\delta \geq \bar{\delta}_\lambda \equiv \frac{v_L - c_L}{\lambda v_L + (1 - \lambda)\lambda - \pi \frac{(1 - \lambda)(1 - \pi)}{\pi}} - c_L.$$
Main results on information aggregation

Theorem 1 (Aggregation Properties)

(i) If (∗) holds strictly, then information aggregation fails along any sequence of equilibria.

(ii) If (∗) does not hold, then there exists a sequence of equilibria along which information aggregates.

Intuition for failure of aggregation:

- Under aggregation, $Q^i_L$ converges to $Q^{i,fict}_L$. But if $Q^{i,fict}_L > v_L$, trade must collapse to zero for large but finite $N$, a contradiction.

- In non-aggregating eqm, $\sigma_N$ declines to zero at rate $N^{-1}$, and the distribution of trades in state $s$ converges to Poisson with mean $\sigma_N \cdot N \cdot \mathbb{P}(\theta_i = L|S = s)$.

But, even if (∗) does not hold, there is no guarantee of information aggregation...
Main results on information aggregation

Theorem 2 (Coexistence)

There exists a $\hat{\delta} < 1$ such that whenever $\delta \in (\hat{\delta}, \bar{\delta}_\lambda)$ and $\lambda$ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails. If $\lambda < \bar{\lambda}$ or $\delta$ is sufficiently small, then information aggregates along all sequences of equilibria.
Main result 1: when does information aggregate?

Option value effect too strong in *dark-shaded* region, but too weak in *unshaded* region; its strength is endogenous to equilibrium played in *light-shaded* region.
Welfare and trade when information aggregates
Welfare and trade when information does not aggregate
Trading behavior and welfare

When information **aggregates**:
- Equilibrium becomes the same as fictitious economy,
- Strategic interactions among sellers vanish,
- Conditional on $S$, uncertainty about trading volume and prices vanishes.

When information **fails to aggregate**:
- Equilibrium different from fictitious economy:
  - $\# \text{ of trades} \mid \text{state } S \sim \text{Poisson with parameter } \sigma N^{\mathbb{P}}(\theta^i = L \mid S)$.
  - Welfare strictly lower than in fictitious economy.
- Strategic interactions **remain**,
- Conditional on $S$, uncertainty about trading volume and prices **remains**.

**Question**: Is info production in the laissez-faire equilibrium efficient?
Simple information policies

Consider a social planner who can control what information and when it is available to agents.

Can she ensure that information aggregates? If so, how? Does she face a tradeoff between aggregation and maximization of trading surplus?

Reporting lags:
- Traders for one asset observe information about other trades with delay.
- Sufficiently long reporting lag reduces incentives to delay trade and ensures that information aggregates (albeit, with delay).
- **But**, a uniform reporting lag alone yields low trading surplus as it does not allow information to mitigate the adverse selection problem.

Segmented platforms:
- Traders observe information in real time only on their platform.
- Size of the platform can be chosen to ensure that reporting lags are not detrimental for welfare, and it may be finite.
Segmented platforms

Welfare and trade when (absent intervention) information does not aggregate
Segmented platforms

Welfare and trade when (absent intervention) information aggregates
Optimal information policy
Optimal information policy

- We suppose that the planner observes trading behavior at $t = 1$ and chooses what information about it to make public at $t = 2$.
  - Application: transparency policies in asset markets.

- Her objective is to maximize expected discounted gains from trade.

- Novel feedback: the planner’s information policy influences trading behavior and, thus, the information content of whatever she communicates.
  - E.g., she cannot choose an informative policy that implies $\sigma = 0$.

- Two-step approach:
  - First, consider problem where planner actually knows state $S$, and use it to obtain an upper bound on the planner’s value in actual problem.
  - Second, construct an information policy that maps observed trades to “messages,” which attains this upper bound as $N \uparrow \infty$. 
Graphical illustration

Partial Revelation is Optimal

Full Revelation is Optimal
Main result 2: normative properties

When laissez-faire is inefficient, optimal policy conceals high state w.p. > 0 in order to weaken the option value effect and accelerate trade (Pareto optimal).
Main result 2: normative properties

Endogenous information constrains policy: when $\lambda$ and $\delta$ are large, the planner would want to reveal the state; but then she would not learn it in the first place!
Conclusions

We study information aggregation properties of dynamic markets with adverse selection and correlated assets.

- We provide necessary and sufficient conditions, under which information aggregation must fail along any sequence of equilibria (LLN fails!).
- If these conditions are violated, there can be a coexistence of aggregating and non-aggregating equilibria.
- Implications for policies that enhance information dissemination in markets:
  - Information design with endogenous information.
  - Reporting lags + segmented trading platforms.
  - Information design: optimality of partial revelation.