Bargaining and News

Brendan Daley
CU Boulder

Brett Green
UC Berkeley

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Motivation

A central issue in the bargaining literature

► Will trade be (inefficiently) delayed?

What is usually ignored

► If trade is in fact delayed, new information may come to light...

This paper = Bargaining + News
A canonical setting

- An indivisible asset (e.g., firm, real estate, security)

- One informed seller and one uninformed buyer
  - Buyer makes price offers
  - Common knowledge of gains from trade
  - Efficient outcome: trade immediately

- Infinite horizon; discounting; frequent offers; no commitment

+ News: information about the asset is gradually revealed
Application 1: Catered Innovation

Consider a startup (the informed seller) that has “catered” its innovation to a large firm, say, Google (the uninformed buyer)

- This exit strategy has become increasingly common (Wang, 2015)
  - Alphabet alone has made over 200 acquisition
  - Nest, Waze, Android, Picasa, YouTube, DropCam

- The longer the startup operates independently, the more Google will learn about the value of the innovation

- But delaying the acquisition is inefficient because Google can leverage economies of scale

Questions:
- How does capacity to learn affect Google's bargaining power?
- How does the exit strategy affect incentives for innovation?
Application 2: Due Diligence

“Large” transactions typically involve a due diligence period:

- Corporate acquisitions
- Commercial real estate transactions

This information gathering stage is inherently dynamic.
- e.g., Verizon’s acquisition of Yahoo

**Questions:** How does the acquirer’s ability to conduct **due diligence** and **renegotiate** the terms
- Initial terms of sale? Eventual terms of sale?
- Profitability of acquisition? Likelihood of deal completion?
The buyer’s ability to extract more surplus is remarkably limited.  
- A negotiation takes place and yet the buyer gains nothing from it.  
- Coasian force overwhelms access to information.

Buyer engages in a form of costly experimentation  
- Makes offers that are sure to lose money if accepted, but generate information if rejected  
- Seller benefits from buyer’s incentive to experiment

Introducing competition can lead to worse outcomes.  
- Under certain conditions, seller’s payoff is higher and/or the outcome is more efficient with a single buyer than with competing ones.
Literature

Bargaining with independent values

Bargaining with interdependent values

News in competitive markets with adverse selection
► Daley and Green (2012), Asriyan, Fuchs and Green (2017)
Model: Players and Values

Players: seller and buyer

- Seller owns asset of type $\theta \in \{L, H\}$
- $\theta$ is the seller’s private information
- Both players are risk neutral, common discount rate $r$

Values:

- Buyer’s value for the asset is $V_\theta$
- Seller’s reservation value is $K_\theta$
- Common knowledge of gains from trade: $V_\theta > K_\theta$
- “Lemons” condition: $K_H > V_L$
Continuous time setting:

- At every $t$ buyer makes offer, $w$, to seller

- If $w$ accepted at time $t$, the payoff to the seller is
  \[ e^{-rt}(w - K_\theta) \]
  and the buyer’s payoff is
  \[ e^{-rt}(V_\theta - w) \]
Suppose $\theta$ is public information.

- The buyer has all the bargaining power.
- The buyer extracts all the surplus.
- Offers $K_\theta$ at $t = 0$ and the seller accepts.

Payoffs:

\[
\text{Buyer payoff} = V_\theta - K_\theta \\
\text{Seller payoff} = 0
\]

Clearly, knowing $\theta$ is good for the buyer.

- What happens if buyer only learns about $\theta$ gradually?
Model: News

- Represented by a **publicly observable** process:

\[ X_t(\omega) = \mu_\theta t + \sigma B_t(\omega) \]

defined on \( \{\Omega, \mathcal{H}, \mathcal{P}\} \) where \( B \) is standard B.M. and \( \mu_H > \mu_L \)

- The **quality of the news** is captured by the signal-to-noise ratio:

\[ \phi \equiv \frac{\mu_H - \mu_L}{\sigma} \]

Buyer makes an offer

Seller accepts (and the game ends) or rejects

News about the seller is revealed

Buyer makes another offer

\[ dt \]
Equilibrium objects

1. Offer process, \( W = \{W_t : 0 \leq t \leq \infty\} \)

2. Seller stopping times: \( \tau^\theta \)
   - Access to private randomization device
   - Endows a CDF over \( \mathcal{H} \)-stopping times: \( \{S_t^\theta : 0 \leq t < \infty\} \)

3. Buyer’s belief process, \( Z = \{Z_t : 0 \leq t \leq \infty\} \)

We look for equilibria that are **stationary** in the buyer’s beliefs:

- \( Z \) is a time-homogenous Markov process
- Offer is a function that depends only on the state, \( W_t = w(Z_t) \)
Buyer’s beliefs

Buyer starts with a prior $P_0 = \Pr(\theta = H)$

- At time $t$, buyer conditions on
  1. the path of the news,
  2. seller rejected all past offers

- Using Bayes Rule, the buyer’s belief at time $t$ is

$$P_t = \frac{P_0 f_t^H(X_t)(1 - S_t^H)}{P_0 f_t^H(X_t)(1 - S_t^H) + (1 - P_0) f_t^L(X_t)(1 - S_t^L)}$$

- Define $Z \equiv \ln \left( \frac{P_t}{1 - P_t} \right)$, we get that

$$Z_t = \ln \left( \frac{P_0}{1 - P_0} \right) + \ln \left( \frac{f_t^H(X_t)}{f_t^L(X_t)} \right) + \ln \left( \frac{1 - S_t^H}{1 - S_t^L} \right)$$
Seller’s problem

Given \((w, Z)\), the seller faces a stopping problem

**Seller’s Problem**

For all \(z\), the seller’s strategy solves

\[
\sup_{\tau} E_z^\theta \left[ e^{-r\tau} (w(Z_\tau) - K_\theta) \right]
\]

Let \(F_\theta(z)\) denote the solution.
Buyer’s problem

In any state $z$, the buyer has essentially three options:

1. **Wait:** Make a non-serious offer that is rejected w.p.1.

2. **Screen:** Make an offer $w < K_H$ that only the low type accepts with positive probability

3. **Buy/Stop:** Offer $w = K_H$ and buy regardless of $\theta$

Let $F_B(z)$ denote the buyer’s value function.
Equilibrium Characterization

Theorem

There exists a unique equilibrium. In it,

- For $P_t \geq b$, trade happens immediately: buyer offers $K_H$ and both type sellers accept
- For $P_t < b$, trade happens “smoothly”: only the low-type seller trades and with probability that is proportional to $dt$.

- i.e., $dQ_t = \dot{q}(Z_t)dt$
Equilibrium: sample path

![Graph showing Belief over Time](image-url)
Equilibrium: sample path
Equilibrium construction

Conjecture the equilibrium is “smooth”

1. Buyer’s problem is linear in the rate of trade: $\dot{q}$
   - Derive $F_B$ (independent of $F_L$)

2. Given $F_B$, what must be true about $F_L$ for smooth trade to be optimal?
   - Derive $F_L$, which implies $w$

3. Low type must be indifferent between waiting and accepting
   - Indifference condition implies $\dot{q}$

Summary: Smooth $\implies F_B \implies F_L \implies \dot{q}$
A bit more about Step 1

\[ rF_B(z) = \frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z) \]

Evolution due to news

\[ + \dot{q}(z) \left( (1 - p(z))(V_L - F_L(z) - F_B(z)) + F'_B(z) \right) \]

\[ \Gamma(z) = \text{net-benefit of screening at } z \]

- Buyer’s value is linear in \( \dot{q} \)
- For “smooth” trade to be optimal, it must be that \( \Gamma(z) = 0 \)
  \[ \rightarrow F_B \text{ does not depend on } \dot{q} \text{ (and has simple closed-form solution) } \]
- Therefore, buyer does not benefit from screening!
  \[ \rightarrow \text{Otherwise, she would want to trade “faster” } \]
  \[ \rightarrow \text{Pins down exactly how expensive it must be to buy } L, \text{ i.e., } F_L(z) \]
Equilibrium payoffs

**Step 1: Buyer value**

**Step 2: Low-type value**
Equilibrium rate of trade

Step 3: Rate of trade, $\dot{q}$

$\dot{q}(b^-) > 0$
Interesting Predictions?

1. Buyer does not benefit from the ability to negotiate the price.
   • Though she *must* negotiate in equilibrium.

2. The buyer is guaranteed to lose money on any offer below $K_H$ that is accepted.
   • A form of costly experimentation.
   • Seller benefits from this behavior.

3. Introducing competition among potential buyers may be both less efficient and worse for the seller.
   • Competition reduces incentive for experimentation.
Who Benefits from the Negotiation?

Suppose the price is **exogenously fixed** at the lowest price that the seller will accept: $K_H$ (e.g., initial terms of sale).

- The buyer conducts due diligence (observes $\hat{Z}$) and decides when and whether to actually complete the deal.
- Buyer’s strategy is simply a stopping rule, where the expected payoff upon stopping in state $z$ is

  $$E_z[V_\theta] - K_H$$

- Call this the **due diligence game**.
  - NB: it is not hard to endogenize the initial terms.
Due Diligence Game

\[ V_H - K_H \]

\[ E[V_\theta] - K_H \]

\[ V_L - K_H \]
Due Diligence Game

\[ V_H - K_H \]

\[ V_L - K_H \]

smooth-pasting

\[ p \]

\[ b \]

\[ 1 \]
Who Benefits from the Negotiation?

Result

In the equilibrium of the bargaining game:

1. The buyer’s payoff is identical to the due diligence game.
2. The \((L\text{-type})\) seller’s payoff is higher than in the due diligence game.

Total surplus higher with bargaining, but fully captured by seller.

- Despite the fact that the buyer makes all the offers.
No Lemons $\implies$ No Learning

\[ V_{H-K_H} \]

\[ E_z[V_\theta-K_H] \]
No Lemons $\implies$ No Learning

\[ F_B = \text{E}[V_\theta] - K_H \]
No Lemons $\implies$ No Learning

**Result**

When $V_L \geq K_H$, unique equilibrium is immediate trade at price $K_H$.

- Absent a lemons condition, the Coasian force overwhelms the buyer’s incentive to learn.
Below $b$, the buyer is making an offer that:

1. will ONLY be accepted by the low type
2. will make a loss whenever accepted

Why?

- One interpretation: costly experimentation
- Buyer willing to lose money today (if offer accepted) in order to learn faster (if rejected)
- The presence of news is necessary for this feature to arise

Caveat: the buyer exhausts all the of the benefits from experimentation leaving her with precisely the same payoff she would obtain if she were unable to experiment.
Relation to the Coase Conjecture

The buyer’s desire to capture future profits from trade leads to a form of intertemporal competition.

- Seller knows buyer will be tempted to increase price tomorrow
- Which increases the price seller is willing to accept today
- Buyer “competes” against future self

**Coase Conjecture:** Absent some form of commitment, the outcome with a monopolistic buyer will resemble the competitive outcome.

**Question:** How does learning/news affect the Coase conjecture?
Competitive equilibrium

Theorem (Daley and Green, 2012)

With competitive buyers, the equilibrium looks as follows:

- For $P_t \geq b$: trade happens immediately, buyers offer $V(P_t)$ and both type sellers accept.
- For $P_t < a$: buyers offer $V_L$, high types reject w.p.1. Low types mix such that the posterior jumps to $a$.
- For $P_t \in (a, b)$: there is no trade, buyers make non-serious offers which are rejected by both types.

- Monopolistic outcome $\neq$ Competitive outcome
Effect of competition

Result

1. Efficient trade requires higher belief with competition: $b_b < b_c$.
2. Competitive equilibrium is strictly less efficient for $p \in (\hat{p}, b_c)$. 
Efficiency
Low-type value
Incentives for Innovation

- **Bilateral**
- **Competitive**
Additional Results

- **Uniqueness**
  - Why trade must be “smooth” below $\beta$

- **The effect of news quality**
  - The no-news limit differs from Deneckere and Liang (2006)

- **Extensions**
  1. Costly investigation
     - Buyer “walks away” when sufficiently pessimistic
     - Seller can be better or worse off
  2. “Lumpy” information arrival
     - Buyer extracts concessions, but experimentation region persists.

**Robust finding**: buyer does not benefit from ability to negotiate.

- Solve analogous due diligence game first ($F_B \implies F_L \implies \dot{q}$)
- Useful heuristic for constructing equilibria with frequent offers
Lumpy Arrivals

$K_H$ - Positive profit from acceptance

Costly experimentation

$V(b_\lambda) - K_H$ - Rejection is bad news

$\frac{\lambda(V_L - K_L)}{\lambda + r}$

$V_L$ - Buyer's Payoff,

$w = F_L$

Buyer's Offer,

$0$ $p_e$ $b_\lambda$

$0$ $V - K_H$

Buyer's Payoff,
Competition for Due Diligence

Suppose there is competition for the right to conduct due diligence.

- Multiple bidders compete in an auction
- The seller selects a winner
- The winner can conduct due diligence and decide whether to complete the transaction, but no price renegotiation

Result

A higher bid is not necessarily better for the seller because it induces stricter due diligence.

- The winning bid lies strictly between $K_H$ and $V_H$
- The winning bidder makes strictly positive profit

To do list:

- Break-up fees, deadlines, equity offers, renegotiation
Implications for Applications

- All else equal, “Catered” innovations will tend to be less profitable business units.

- A downward revision of the price during due diligence is **bad news** for the acquirer.
  - Acquirer’s stock prices should fall in response.
  - E.g., when Verizon announced acquisition of Yahoo to go through at price $300M less than originally specified.

- A target firm will not (and should not) necessarily accept the highest bid from potential acquirers.
We explore the effect of news in a canonical bargaining environment

- Construct the equilibrium (in closed form).

- Buyer’s ability to leverage news to extract surplus is remarkably limited.
  - Buyer negotiates based on new information in equilibrium, but gains nothing from doing so.
  - The robust implication of the Coasian force

- Relation to the competitive outcome
  - Competition eliminates the Coasian force, may reduce both total surplus and seller payoff.
  - But competition also provides stronger incentives for innovation.
Other equilibria?

We focused on the (unique) smooth equilibrium. Can other stationary equilibria exist?

▷ No

By Lesbegue's decomposition theorem for monotonic functions

\[ Q = Q_{abs} + Q_{jump} + Q_{singular} \]

To sketch the argument, we will illustrate how to rule out:

1. Atoms of trade with \( L \) (i.e., \( Q_{jump} = 0 \))
2. Reflecting barriers (i.e., \( Q_{singular} = 0 \))
Uniqueness

Suppose there is some $z_0$ such that:

- Buyer makes offer $w_0$
- Low type accepts with atom

Let $\alpha$ denote the buyer’s belief conditional on a rejection. Then

1. $F_L(z_0) = F_L(\alpha) = w_0$, by seller optimality
2. $F_L(z) = w_0$ for all $z \in (z_0, \alpha)$, by buyer optimality

Therefore, starting from any $z \in (z_0, \alpha)$, the belief conditional on a rejection jumps to $\alpha$.

- If there is an atom, the behavior must resemble the competitive-buyer model...
Why trade must be smooth with a single buyer

\[ F_L = w \text{ below } \alpha \]

\[ F_L \text{ implied by proposed strategies} \]
Why trade must be smooth with a single buyer

Intuitively,

- \( L \) is no more expensive to trade with at \( z = \alpha + \epsilon \) than at \( z = \alpha \).
- If the buyer wants to trade with \( L \) at price \( w \) below \( z = \alpha \), he will want to extend this behavior above \( z = \alpha \) as well.
Proposition (The effect of news quality)

As the quality of news increases:

1. Both $\beta$ and $F_B$ increase

2. The rate of trade, $\dot{q}$, decreases for low beliefs but increases for intermediate beliefs

3. Total surplus and $F_L$ increase for low beliefs, but decrease for intermediate beliefs

Two opposing forces driving 3.

- Higher $\phi$ increases volatility of $\hat{Z} \implies$ faster trade
- Higher $\beta$ (and/or) lower $\dot{q} \implies$ slower trade
Effect of news on buyer payoff
Effect of news on buyer payoff

\[ V_H - K_H \]

\[ p \]
Effect of news on buyer payoff

The graph shows the relationship between $V_H - K_H$ and $p$. The curves represent different values of $b_1$, $b_2$, and $b_3$. The vertical axis represents $V_H - K_H$, and the horizontal axis represents $p$. The graph illustrates how the payoff changes with different values of $p$.
Effect of news on buyer payoff
Effect of news on low-type payoff
Effect of news on low-type payoff

\[ K_H \]

\[ V_L \]

\[ b_1 \quad b_2 \quad b_3 \quad 1 \]

0
(In)efficiency

% Loss

\( p \)

\( b_1 \)

\( b_2 \)

\( b_3 \)
As news quality becomes arbitrarily high ($\phi \to \infty$):

1. $\beta \to \infty$ (i.e., $b \to 1$)
2. $F_B \xrightarrow{u} p(z)(V_H - K_H)$
3. $F_L \xrightarrow{pw} V_L$
4. $\dot{q} \xrightarrow{pw} \infty$

Note that buyer waits until certain that $\theta = H$ before offering $K_H$

- Captures full surplus from trade with high type
- But NONE of the surplus from trade with low type
Arbitrarily low quality news

Result

As news quality becomes arbitrarily low ($\phi \rightarrow 0$):

1. $\beta \rightarrow \bar{z}$

2. $F_B \xrightarrow{u} \max\{0, V(z) - K_H\}$

3. $F_L \xrightarrow{pw}\begin{cases} V_L & \text{if } z < \bar{z} \\ \frac{e-1}{e} V_L + \frac{1}{e} K_H & \text{if } z = \bar{z} \\ K_H & \text{if } z > \bar{z} \end{cases}$

4. for all $z < \bar{z}$, $\dot{q}(z) \rightarrow \infty$, but $\dot{q}(\bar{z}) \rightarrow 0$
Limiting payoffs

\[ V_{II} - K_{II} \]

\[ K_H \]

\[ V_L \]

Buyer payoff

Low type payoff
Effect of news

Our $\phi \to 0$ limit differs from Deneckere and Liang (2006)
Effect of news

Intuition for DL06:

- Coasian force disappears at precisely $Z_t = z$
- Buyer leverages this to extract concessions from low type at $z < \bar{z}$
Effect of news

With news, his belief cannot just “sit at $z$”, so this power evaporates.

- Even with arbitrarily low-quality news!
With news, his belief cannot just “sit at $z$”, so this power evaporates.

▶ Even with arbitrarily low-quality news!
Stochastic control problem

The buyer must decide:

- How quickly to trade with only the low type (i.e., choose $Q$ given $F_L$)
- When to “buy the market” (i.e., choose $T$ at which to offer $K_H$)

**Buyer’s Problem**

Choose $(Q, T)$ to solve, for all $z$,

$$
\sup_{Q,T} \left\{ (1 - p(z))E_z^L \left[ \int_0^T e^{-rt}(V_L - F_L(\hat{Z}_t + Q_t))e^{-Q_t} dQ_t 
+ e^{-(rT+QT)}(V_L - K_H) \right] + p(z)E_z^H \left[ e^{-rT}(V_H - K_H) \right] \right\}
$$

Let $F_B(z)$ denote the solution.
Buyer’s problem

Lemma

For all $z$, $F_B(z)$ satisfies:

**Option to wait:**

$$r F_B(z) \geq \frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)$$

**Optimal screening:**

$$F_B(z) \geq \sup_{z' > z} \left\{ \left(1 - \frac{p(z)}{p(z')}\right) (V_L - F_L(z')) + \frac{p(z)}{p(z')} F_B(z') \right\}$$

**Option to buy:**

$$F_B(z) \geq E_z[V_\theta] - K_H$$

where at least one of the inequalities must hold with equality.
Equilibrium construction

1. For $z < \beta$, $w(z) = F_L(z)$ and the buyer’s value is

$$F_B(z) = (V_L - F_L(z)) (1 - p(z)) \dot{q}(z) dt + \left(1 - \frac{\dot{q}(z)}{1 + e^z} dt\right) E_z [F_B(z + dZ_t)]$$

and $dZ_t = d\hat{Z}_t + \dot{q}(Z_t) dt$. So,

$$rF_B(z) = \frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)$$

\text{Evolution due to news}

$$+ \dot{q}(z) \left((1 - p(z))(V_L - F_L(z) - F_B(z)) + F'_B(z)\right)$$

$\Gamma(z)=$net-benefit of screening at $z$
Equilibrium construction

2. Observe that the buyer’s problem is linear in \( \dot{q} \)

\[
r F_B(z) = \frac{\phi^2}{2} (2p - 1) F'_B + \frac{\phi^2}{2} F''_B
\]

Evolution due to news

\[
+ \sup_{\dot{q} \geq 0} \dot{q} \left( (1 - p)(V_L - F_L - F_B) + F'_B \right)
\]

\( \Gamma(z) = \text{net-benefit of screening} \)

Hence, in any state \( z < \beta \), either

(i) the buyer strictly prefers \( \dot{q} = 0 \), or
(ii) the buyer is indifferent over all \( \dot{q} \in \mathbb{R}_+ \)
3. In either case

\[ \dot{q}(z) \Gamma(z) = 0 \]

4. This simplifies the ODE for \( F_B \) to just

\[ rF_B = \frac{\phi^2}{2} (2p - 1) F'_B + \frac{\phi^2}{2} F''_B \]

→ \( F_B \) does not depend on \( \dot{q} \)

→ Buyer gets same value he would get from \( \dot{q} = 0 \)

→ Buyer gains nothing from the ability to screen using prices!
Equilibrium construction

Using the appropriate boundary conditions, we find \( F_B(z) = C_1 \frac{e^{u_1 z}}{1 + e^z} \),

where \( u_1 = \frac{1}{2} \left( 1 + \sqrt{1 + 8r/\phi^2} \right) \) and \( C_1 \) solves VM and SP at \( z = \beta \).
Next, conjecture that $\dot{q}(z) > 0$ for all $z < \beta$. Then, it must be that

$$\Gamma(z) = 0$$

Or equivalently

$$F_L(z) = (1 + e^z)F_B'(z) + V_L - F_B(z)$$

This pins down exactly how “expensive” the low type must be for the buyer to be indifferent to the speed of trade (i.e., $F_L$).
Equilibrium construction

For \( z < \beta \), the low-type must be indifferent between accepting \( w(z) \) and waiting.

The waiting payoff is

\[
F_L(z) = \mathbb{E}_Z^L \left[ e^{-rT(\beta)} K_H \right]
\]

which evolves as

\[
r F_L(z) = \left( \dot{q}(z) - \frac{\phi^2}{2} \right) F'_L(z) + \frac{\phi^2}{2} F''_L(z)
\]

So, \( \dot{q}(z) \) must satisfy

\[
\dot{q}(z) = \frac{r F_L(z) + \frac{\phi^2}{2} F'_L(z) - \frac{\phi^2}{2} F''_L(z)}{F'_L(z)}
\]