Liquidity Sentiments

Vladimir Asriyan  William Fuchs  Brett Green
CREI  UT Austin and UC3M  UC Berkeley

March 2019
Introduction

Asset markets exhibit time variation in “liquidity”

- E.g., real estate, MBS, repo, merger waves, “physical” capital
- Liquidity is procyclical, positively correlated with prices
  - e.g., liquidity dries up in bad times
- The volatility in liquidity and prices often appears unrelated to new information or shocks to fundamentals
  - Usually interpreted as a ‘behavioral’ phenomenon: irrational exuberance, animal spirits, overconfidence, sentiments...
Asset markets exhibit time variation in “liquidity”

- E.g., real estate, MBS, repo, merger waves, “physical” capital
- Liquidity is procyclical, positively correlated with prices
  - e.g., liquidity dries up in bad times
- The volatility in liquidity and prices often appears unrelated to new information or shocks to fundamentals
  - Usually interpreted as a ‘behavioral’ phenomenon: irrational exuberance, animal spirits, overconfidence, sentiments...

**Questions:** Is there a fundamental link between prices and liquidity within a rational framework? Is there a role for “sentiments”? 
Our (hopefully) non-controversial starting point

- The efficient owner of an asset may vary over time
  - Capital should be reallocated to the most productive firms
  - Real estate transacts due to life cycle, labor market shocks, etc.

- Trade is the consequence of the emergence of gains from trade

- Liquidity – ease with which these gains are realized – is therefore an intrinsic determinant of “fundamental” value

- Without frictions, all gains are realized immediately.
  - Assets always held by those who value them the most.
Our (hopefully) non-controversial starting point

- The **efficient owner** of an asset may vary over time
  - Capital should be reallocated to the most productive firms
  - Real estate transacts due to life cycle, labor market shocks, etc.

- Trade is the consequence of the emergence of gains from trade

- **Liquidity** – ease with which these gains are realized – is therefore an intrinsic determinant of “fundamental” value

- Without frictions, all gains are realized immediately.
  - Assets always held by those who value them the most.

- **Information frictions** can hinder liquidity.
What we do

Analyze a model with asymmetric information and resale considerations

► Buyers worry about:

1. **Quality** of assets for which they compete, and
2. **Liquidity** they will face when trying to resell in the future.

► We show that **intertemporal complementarities** emerge

* If buyers expect a **liquid market tomorrow**
  * They are willing to bid more aggressively for the assets today
  * Quality of assets that sellers willing to trade improves
  * Which leads to **high liquidity and high prices today**
The intertemporal coordination problem generates multiple self-fulfilling equilibria

- **Sentiments**: defined as expectations about future market conditions, generate endogenous volatility
  - The model disciplines set of equilibrium sentiment dynamics
  - Sentiments must be stochastic and sufficiently persistent
The intertemporal coordination problem generates multiple self-fulfilling equilibria.

- **Sentiments**: defined as expectations about future market conditions, generate endogenous volatility
  - The model disciplines set of equilibrium sentiment dynamics
  - Sentiments must be stochastic and sufficiently persistent

- With endogenous asset production (and moderate production costs)
  - Sentiments are a necessary part of any equilibrium
  - Quality produced is higher in “bad” times
Applications

- Capital reallocation among firms
  - Reallocation is procyclical
  - Productivity dispersion can go either way
  - Aggregate productivity depends on sentiments

- Real estate boom/bust cycles
  - Strong sentiments: high prices, high turnover, low time-to-sale.
  - Weak sentiments: low prices, low turnover, high time-to-sale.
Related literature

  - **Coordination (static)**: Plantin (2009), Malherbe (2014)


Model

Discrete time, infinite horizon, \( t = 0, 1, 2, \ldots \).

**Assets:** Unit mass of assets indexed by \( i \in [0, 1] \)

- Asset \( i \) has (fixed) quality \( \theta_i \in \{ L, H \} \)
- Fraction \( \pi \) of assets are high quality
Model

Discrete time, infinite horizon, \( t = 0, 1, 2, \ldots \).

Assets: Unit mass of assets indexed by \( i \in [0, 1] \)

- Asset \( i \) has (fixed) quality \( \theta_i \in \{L, H\} \)
- Fraction \( \pi \) of assets are high quality

Agents: Mass \( M \gg 1 \) of agents, indexed by \( j \in [0, M] \)

- Agents are risk-neutral with common discount factor \( \delta \)
- Each agent can hold at most one unit of the asset
- Agent \( j \) at time \( t \) has private value or productivity \( \omega_{j,t} \in \{l, h\} \)
- Productivity is iid with \( \lambda = P(\omega_{jt} = l) \)
Flow Payoffs

If agent $j$ owns asset $i$ at date $t$:

- She receives a flow payoff $x_{ijt} = u(\theta_i, \omega_{jt})$
- High quality assets deliver higher payoff, $u(H, \omega) > u(L, \omega)$
- More productive agents generate higher payoff

\[
v_\theta \equiv u(\theta, h) > c_\theta \equiv u(\theta, l)
\]

Gains from trade exist
Markets

Asset markets are competitive and decentralized. In each period:

- Multiple productive buyers bid for each asset à la Bertrand.
- Seller can accept an offer or reject and wait until the next period.
  - Buyer whose offer is accepted becomes asset owner
  - Owner who sells an asset becomes a buyer next period
Information friction

Absent frictions, outcome is efficient.

- Markets would reallocate assets from unproductive owners to productive non-owners (buyers).
Information friction

Absent frictions, outcome is efficient.

- Markets would reallocate assets from unproductive owners to productive non-owners (buyers).

But there is asymmetric information:

- Owner privately observes asset quality and productivity, \((\theta, \omega)\).
- Trading is anonymous
  - History of asset or owner transactions is not observable
  - Rules out signaling through delay
Equilibrium concept

We look for Stationary Rational Expectations Equilibria. This has three main requirements:

- **Owner optimality.** Each owner makes her selling decisions optimally, taking as given the strategies of all other agents.

- **Buyer optimality.** Each buyer makes her bidding decision optimally, given her beliefs and the strategies of other buyers.

- **Belief consistency.** Buyer’s beliefs about future play and who trades today are consistent with the equilibrium strategies.
Benchmark without information frictions

Result

If asset qualities are observable, then the equilibirum is unique. In it,

- All assets are allocated efficiently,
- For all $t$, the price of a type-$\theta$ asset is

$$p_{\theta} = \frac{v_{\theta}}{1 - \delta}$$

and total output is

$$Y^{FB} = \int_{i} v_{\theta} di = E\{v_{\theta}\}$$
Benchmark without information frictions

Result

If asset qualities are observable, then the equilibrium is unique. In it,

- All assets are allocated efficiently,
- For all $t$, the price of a type-$\theta$ asset is

$$p_\theta = \frac{v_\theta}{1 - \delta}$$

and total output is

$$Y_{FB} = \int v_\theta di = E\{v_\theta\}$$

- How do information frictions change this picture?
Stationary equilibrium

First characterize stationary equilibria in which the price is constant, $p^*$. 
First characterize stationary equilibria in which the price is constant, $p^*$. 

1. **Owner optimality.**
   
   - A $(\theta, \omega)$-owner’s value function satisfies:
     
     $$V^*(\theta, \omega) = \max \{ p^*, u(\theta, \omega) + \delta E\{V^*(\theta, \omega')\} \}$$
   
   - The set of owner types who optimally accept a (maximal) offer $p$ is:
     
     $$\Gamma(p) = \{(\theta, \omega) : u(\theta, \omega) + \delta V^*(\theta, \omega') \leq p\}$$
Stationary equilibrium

2. Buyer Optimality

• Bertrand competition among buyers $\implies$ zero profit

$$p^* = E\{v_\theta + \delta V^*(\theta, \omega')|((\theta, \omega) \in \Gamma(p^*))\}.$$  

• No profitable deviation for buyers $\iff$ for all $p \geq p^*$

$$p \geq E\{v_\theta + \delta V^*(\theta, \omega')|((\theta, \omega) \in \Gamma(p))\}.$$
Characterization of Stationary Equilibria

Result

In any stationary equilibrium,

\[ V^*(L, l) = V^*(L, h) = p^* \leq V^*(H, l) < V^*(H, h). \]

Thus, \((L, l)\)-owners always trade, whereas \((H, h)\)-owners never do.

- Two candidate stationary equilibria, depending on whether 
  \((H, l)\)-owner trades.
Candidate stationary equilibria

Efficient trade equilibrium: \((H, l)\)-owner trades

- All gains from trade are realized, prices and total output are:

\[
p^{ET} = V^{ET}(H, l) \quad Y^{ET} = E\{v_\theta\}
\]
Candidate stationary equilibria

Efficient trade equilibrium: \((H, l)\)-owner trades

- All gains from trade are realized, prices and total output are:

\[
p^{ET} = V^{ET}(H, l) \quad Y^{ET} = E\{v_\theta\}
\]

Inefficient trade equilibrium: \((H, l)\)-owner does not trade

- Some gains from trade are unrealized, prices and total output are:

\[
p^{IT} < V^{IT}(H, l) \quad Y^{IT} = E\{v_\theta\} - \lambda\pi(v_H - c_H)
\]

loss from misallocation
There exists two thresholds $\pi < \bar{\pi}$ such that:

1. Efficient trade is an equilibrium iff $\pi \geq \pi$,
2. Inefficient trade is an equilibrium iff $\pi \leq \bar{\pi}$.

Notably, both equilibria exist for $\pi \in (\underline{\pi}, \bar{\pi})$. 

**Theorem**
There exists two thresholds $\underline{\pi} < \bar{\pi}$ such that:

1. Efficient trade is an equilibrium iff $\pi \geq \underline{\pi}$,

2. Inefficient trade is an equilibrium iff $\pi \leq \bar{\pi}$.

Notably, both equilibria exist for $\pi \in (\underline{\pi}, \bar{\pi})$.

- Dynamic considerations are crucial for multiplicity.
Multiplicity and the role of dynamics
What is the source of multiplicity?

An **intertemporal coordination problem:**

- If buyers today expect future markets to be illiquid.
  - Their unconditional value today for an asset is low.
  - Hence the highest (pooling) price they are willing to offer is low.
  - At this low offer, the \((H, l)\)-owners prefer to hold.

- Conversely, if buyers today expect future markets to be liquid.
  - Their unconditional value today for an asset is high.
  - Hence they are willing offer a high (pooling) price.
  - At this high price, the \((H, l)\)-owners are willing to sell.
What is the source of multiplicity?

Efficient trade. Must be that \((H, l)\)-owner does not want to reject:

\[
V^{ET}(H, l) = p^{ET} \geq c_H + \delta E\{V^{ET}(H, \omega')\}
\]

\[
\hat{\pi}v_H + (1 - \hat{\pi})v_L - c_H \geq \delta(1 - \hat{\pi})E\{V^{ET}(H, \omega) - V^{ET}(L, \omega)\}
\]

today’s gain from selling \quad \text{future loss from selling at low price}
What is the source of multiplicity?

**Efficient trade.** Must be that \((H, l)\)-owner does not want to reject:

\[
V^{ET}(H, l) = p^{ET} \geq c_H + \delta E\{V^{ET}(H, \omega')\}
\]

\[
\hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H \geq \delta (1 - \hat{\pi}) E\{V^{ET}(H, \omega) - V^{ET}(L, \omega)\}
\]

\(\Delta^{ET}\)

\(\text{today's gain from selling}\)

\(\text{future loss from selling at low price}\)

**Inefficient trade.** Sufficient to check that buyers do not want to deviate:

\[
V^{IT}(H, l) \geq \hat{\pi} V^{IT}(H, h) + (1 - \hat{\pi}) \left( v_L + \delta E\{V^{IT}(L, \omega')\} \right)
\]

\[
\hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H \leq \delta (1 - \hat{\pi}) E\{V^{IT}(H, \omega) - V^{IT}(L, \omega)\}
\]

\(\Delta^{IT}\)

\(\text{today's gain from buying}\)

\(\text{future loss from buying at high price}\)
What is the source of multiplicity?

You might have noticed that it is actually the same condition, with the inequality reversed.
What is the source of multiplicity?

You might have noticed that it is actually the same condition, with the inequality reversed...but

$$\Delta^{IT} > \Delta^{ET}$$

- High quality assets are relatively more valuable when assets are harder to trade.
Are there other equilibria?

**Result**

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.
Are there other equilibria?

Result

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.

Intuition?

- Suppose trade efficient at $t + 1$ but inefficient at $t$
Are there other equilibria?

**Result**

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.

**Intuition?**

- Suppose trade efficient at $t + 1$ but inefficient at $t$
- Then future market conditions are weakly better at $t$ than at $t + 1$
- Hence trade must also be efficient at $t$
Sentiment equilibrium

- Let $z_t$ denote a publicly observable stochastic process.
- An equilibrium is said to be a sentiment equilibrium with sunspot $z_t$ if prices and allocations depend on its realization.
Let $z_t$ denote a publicly observable stochastic process.

An equilibrium is said to be a sentiment equilibrium with sunspot $z_t$ if prices and allocations depend on its realization.

Let’s begin with a simple Markov family

- **Binary:** $z_t \in \{B, G\}$.
- **Symmetric:** $\rho = \mathbb{P}(z_{t+1} = B | z_t = B) = \mathbb{P}(z_{t+1} = G | z_t = G)$.
- **Candidate Equilibrium:** play efficient trade iff $z_t = G$. 

When does such a sentiment equilibrium exist and what are its properties?
Let $z_t$ denote a publicly observable stochastic process.

An equilibrium is said to be a sentiment equilibrium with sunspot $z_t$ if prices and allocations depend on its realization.

Let’s begin with a simple Markov family

- **Binary:** $z_t \in \{B, G\}$.
- **Symmetric:** $\rho = \mathbb{P}(z_{t+1} = B|z_t = B) = \mathbb{P}(z_{t+1} = G|z_t = G)$.
- Candidate Equilibrium: play efficient trade iff $z_t = G$.

When does such a sentiment equilibrium exist and what are its properties?
A simple class

Result

A sentiment equilibrium with a binary-symmetric first-order Markov sentiment process $z_t$ exists if and only if $\pi \in (\underline{\pi}, \bar{\pi})$ and $\rho \geq \bar{\rho}$, where $\bar{\rho}$ depends on parameters.
A simple class

Result

A sentiment equilibrium with a binary-symmetric first-order Markov sentiment process $z_t$ exists if and only if $\pi \in (\bar{\pi}, \pi)$ and $\rho \geq \bar{\rho}$, where $\bar{\rho}$ depends on parameters.

- Not anything goes!
  - Sentiments needs to be sufficiently persistent to facilitate intertemporal coordination.

- It needs to signal to agents:
  - How to behave today
  - That liquidity is likely to be similar in the future

- Otherwise, profitable deviations exist!
When do Sentiment equilibria exist?

![Graph showing the conditions for different types of trade equilibria based on the proportion of high quality assets and the discount factor. The graph is divided into three regions: Only Efficient Trade Equilibrium Exists, Both Equilibria Exist, and Only Inefficient Trade Equilibrium Exists.]
When do Sentiment equilibria exist?

The diagram illustrates the relationship between the proportion of high quality assets and the discount factor. It shows three regions:

1. **Only Efficient Trade Equilibrium Exists**
2. **Sentiments Exist**
3. **Only Inefficient Trade Equilibrium Exists**

The lines on the graph represent different scenarios based on the values of the discount factor and the proportion of high quality assets.
When do Sentiment equilibria exist?

![Graph showing the relationship between sunspot persistence and the proportion of high quality assets. The shaded area indicates the range where sentiments exist.](image-url)
Sentiments can be richer...

Example

- Sunspot process: Markov chain $z_t \in \{1, ..., N\}$

- Transition matrix: $Q$

\[
Q = \begin{bmatrix}
\rho & 1 - \rho & 0 & \ldots & 0 \\
\frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} & \ldots & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ldots & \frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} \\
0 & 0 & \ldots & 1 - \rho & \rho \\
\end{bmatrix}
\]

- Candidate Equilibrium: play efficient trade iff $z_t \geq n^* \in \{1, N\}$
Sentiments can be richer...

Figure: $N = 40$, $n^* = 20$, $\rho = 0.4$
Sentiments can be richer...

Figure: $N = 40$, $n^* = 20$, $\rho = 0.4$
Sentiments can be richer...

Figure: $N = 40, n^* = 20, \rho = 0.4$
Going beyond the simple family

Theorem (Sentiments)

A sentiment equilibrium with a Markov sunspot $z_t$ exists if and only if $\pi \in (\underline{\pi}, \bar{\pi})$ and the equilibrium play it supports is sufficiently “persistent”

- Formal notion of sufficiently persistent provided in the paper
- Intuition is similar to before: to induce liquidity today, must be sufficiently likely that market will remain liquid tomorrow.
Production

Thus far, distribution of asset quality was exogenous.

▶ Suppose that each period, a mass of producers can create an asset.
▶ In period \( t \), each producer chooses how much to invest.
  • Choose investment level \( q \) at cost \( c(q) \), with \( c' > 0, c'' \geq 0 \)
  • Produces \( H \) quality asset w.p. \( q \) (and \( L \) quality otherwise)
▶ In period \( t + 1 \), the producer becomes the owner of the asset.
Thus far, distribution of asset quality was exogenous.

- Suppose that each period, a mass of producers can create an asset.
- In period $t$, each producer chooses how much to invest.
  - Choose investment level $q$ at cost $c(q)$, with $c' > 0$, $c'' \geq 0$
  - Produces $H$ quality asset w.p. $q$ (and $L$ quality otherwise)
- In period $t + 1$, the producer becomes the owner of the asset.

For simplicity, we will assume that:

- Asset vintage is observable.
  - Avoids constructing equilibria with time-varying distribution of assets.
- Producer $\omega$ iid and same distribution as agents.
First Order Condition

Date-$t$ producer chooses $q$ to solve

$$\max_{q \in [0,1]} \left\{ \delta \left( q E_t \{ V_{t+1}^* (H, \omega) \} + (1 - q) E_t \{ V_{t+1}^* (L, \omega) \} \right) - c(q) \right\}$$
First Order Condition

Date-\(t\) producer chooses \(q\) to solve

\[
\max_{q \in [0,1]} \left\{ \delta \left( qE_t\{V_{t+1}(H, \omega)\} + (1 - q)E_t\{V_{t+1}(L, \omega)\} \right) - c(q) \right\}
\]

The FOC for investment at time \(t\) is

\[
c'(q_t) = \delta \left( E_t\{V_{t+1}^*(H, \omega) - V_{t+1}^*(L, \omega)\} \right) \Delta_{t+1}^*
\]

And \(\Delta_{t+1}^*\) is lower when liquidity sentiments are higher (e.g., \(z_t = G\)).

\[\text{Implication: If a sentiment equilibrium exists, then lower quality assets will be produced in “good” times.}\]
First Order Condition

Date-$t$ producer chooses $q$ to solve

$$\max_{q \in [0,1]} \left\{ \delta \left( qE_t\{V^*_{t+1}(H, \omega)\} + (1 - q)E_t\{V^*_{t+1}(L, \omega)\} \right) - c(q) \right\}$$

The FOC for investment at time $t$ is

$$c'(q_t) = \delta \left( E_t\{V^*_{t+1}(H, \omega) - V^*_{t+1}(L, \omega)\} \right)$$

And $\Delta^*$ is lower when liquidity sentiments are higher (e.g., $z_t = G$)

- **Implication**: If a sentiment equilibrium exists, then lower quality assets will be produced in “good” times.
Sentiments with endogenous production?

Result

When asset production is endogenous:

- Efficient trade is an equilibrium $\iff c'(\pi) \leq c \equiv \Delta_{ET}(\pi)$
- Inefficient trade is an equilibrium $\iff c'({\bar{\pi}}) \geq {\bar{c}} \equiv \Delta_{IT}({\bar{\pi}})$
Sentiments with endogenous production?

When asset production is endogenous:

- Efficient trade is an equilibrium ≜ $c'(\pi) \leq c \equiv \Delta_{ET}(\pi)$
- Inefficient trade is an equilibrium ≜ $c'(\bar{\pi}) \geq \bar{c} \equiv \Delta_{IT}(\bar{\pi})$

Otherwise, any equilibrium must involve sentiments (and a sentiment equilibrium exists).
Illustrating the Result

Only Sentiment Equilibria Exist

Marginal cost of production, $c'(q)$

Level of investment, $q$

$\Delta^{IT}(q)$

$\Delta^{ET}(q)$
What elements of the model are crucial?

1. Informational environment
   - Need asymmetric information about common value component, $\theta$
   - Asymmetric information about $\omega$ not crucial

2. Competition
   - Similar conditions under which sentiments exist with single buyer.

3. Asset quality
   - Need some persistence in quality and some durability.
What elements of the model are crucial?

1. Informational environment
   - Need asymmetric information about common value component, \( \theta \)
   - Asymmetric information about \( \omega \) not crucial

2. Competition
   - Similar conditions under which sentiments exist with single buyer.

3. Asset quality
   - Need some persistence in quality and some durability.

4. Non iid productivity shocks \( \implies \) market history matters
   - Deterministic liquidity cycles can exist (Chiu and Koeppel 2016)
   - Positive autocorrelation: higher liquidity in the past implies lower liquidity today.
Applications of Sentiments: Capital Reallocation

- Agents are firms, $\omega_j$ is firm $j$ productivity
- Assets are capital, $\theta_i$ is quality of capital unit $i$
- Firm $j$’s output and productivity $= u(\theta, \omega_j)$
- Total output and productivity $= \int u(\theta_i, \omega_j) di$
- Trade corresponds to reallocating capital to more productive firm
Applications of Sentiments: Capital Reallocation

- Agents are firms, $\omega_j$ is firm $j$ productivity
- Assets are capital, $\theta_i$ is quality of capital unit $i$
- Firm $j$’s output and productivity $= u(\theta, \omega_j)$
- Total output and productivity $= \int u(\theta_i, \omega_j) d\theta$
- Trade corresponds to reallocating capital to more productive firm

Predictions

- Good times ($z_t = G$): higher output and productivity, only efficient firms operate capital, higher rates of capital reallocation.
- Bad times ($z_t = B$): lower output and productivity, some inefficient firms operate, lower rate of capital reallocation.
Applications of Sentiments: Real Estate

- Agents are households, $\omega_j$ is private value of ownership
- Assets are houses, $\theta_i$ is unobservable quality of house $i$
- Flow payoff to household $j$ from ownership $= u(\theta, \omega_j)$
- Trade corresponds to selling house to higher private value HH
Applications of Sentiments: Real Estate

- Agents are households, $\omega_j$ is private value of ownership
- Assets are houses, $\theta_i$ is unobservable quality of house $i$
- Flow payoff to household $j$ from ownership $= u(\theta, \omega_j)$
- Trade corresponds to selling house to higher private value HH

Predictions

- Boom ($z_t = G$): high prices and volume, low time on the market.
- Bust ($z_t = B$): low prices and volume, high time on the market.
Conclusions

- Adverse selection + resale considerations leads to an inter-temporal coordination problem:
  - Multiple self-fulfilling equilibria exist.

- **Sentiments**: expectations about future market conditions, generate endogenous volatility in prices, liquidity, output, etc.
  - The model disciplines set of possible sentiment dynamics.
  - Must be stochastic and sufficiently persistent.

- With endogenous asset production:
  - Sentiments are necessary for intermediate production costs.
  - Quality of assets produced is better in “bad” times.

- Applications to capital reallocation and real estate markets.